Antiderivative

Goal ~> Given $f(x)$, find $F(x)$ so $F'(x) = f(x)$.

If $F'(x) = G'(x)$ on (a,b), $F(x) = G(x) + C$ on (a,b)
(by MVT)

e.g.) $\sqrt{x} = f$ $\quad$ $f = 1/x$ $\quad$ $f = \frac{1}{1+x^2}$

$F = \frac{2}{3}x^{3/2}$ $\quad$ $F = \ln|x|$ $\quad$ $F = \arctan(x)$

Problems.

Find Antid. for

1.) $(x^2+1)^2$ $\quad (= x^4 + 2x^2 + 1 \rightsquigarrow \frac{1}{5}x^5 + \frac{2}{3}x^3 + x)$

2.) $8^x + x^{-4/5}$ $\rightsquigarrow \frac{1}{\ln(8)} 8^x + 5x^{1/5}$

3.) $\dfrac{e^{4x} + 4^x}{e^2} \rightsquigarrow \frac{1}{e^2}\left(\frac{1}{4}e^{4x} + \frac{1}{\ln(4)}4^x\right)$

4.) $f''(t) = t^2 + 1/t^2$; $f(2) = 3$; $f'(1) = 2$ $\quad$ Find f.
$t > 0$

$f'(t) = \frac{1}{3}t^3 - 1/t + C_1$

$f(t) = \frac{1}{12}t^4 - \ln|t| + \frac{8}{3}t + C_2$

$f'(1) = 2 \to \frac{1}{3} - 1 + C_1 = 2$

$f(2) = 3 \to 16/12 - \ln(2) + 16/3 + C_2 = 3$

$3 - 20/3 + \ln(2) = C_2$

$\ln(2) - 11/3$

Solutions

Math 1A Spring 2025 Quiz 7

Name:

1. Use linear approximation to estimate $\cos 29°$.

$\cos(\frac{\pi}{6} - \frac{\pi}{180}) \approx (-\sin(\pi/6))(-\frac{\pi}{180}) + \cos(\pi/6)$

$= (-1/2)(-\pi/180) + \sqrt{3}/2 = \sqrt{3}/2 + \pi/360$

2. Find the absolute maximum and absolute minimum values of $f(t) = \sqrt{t}/(1+t^2)$ on the interval $[0,2]$.

$f'(t) = \frac{(1+t^2)(\frac{1}{2\sqrt{t}}) - \sqrt{t}(2t)}{(1+t^2)^2} = \frac{1-3t^2}{2\sqrt{t}(1+t^2)^2}$

$t = \pm 1/\sqrt{3}$

t	$f(t)$	
0	0	- min
$1/\sqrt{3}$	$\frac{3}{\sqrt[4]{3}\cdot 4}$	- max.
2	$\sqrt{2}/5$	

3. Let $G(x) = 5x^{2/3} - 2x^{5/3}$.

   (a) Find the intervals of increase or decrease.

   (b) Find the local maximum or minimum values.

   (c) Find the intervals of concavity and inflection points.

A.) $G'(x) = \frac{10}{3}x^{-1/3} - \frac{10}{3}x^{2/3}$

$= (\frac{10}{3})(x^{2/3})(\frac{1}{x} - 1)$

Pos. on $0 < x < 1$

Neg. on $x < 0$, $x > 1$

B.) local min $x = 0$

local max $x = 1$

C.) $G''(x) = -\frac{10}{9}x^{-4/3} - \frac{20}{9}x^{-1/3}$

$= (-\frac{10}{9})(x^{-4/3})(1-x)$

Conc. Down $x < 1$, $x \neq 0$

Conc. up $x > 1$

Infl. $x = 1$

4. Use l'Hospital's Rule to solve the following limit: $\lim_{x\to\infty} x^{e^{-x}}$.

$= \exp(\lim_{x\to\infty} e^{-x}\ln(x)) = \exp(\lim_{x\to\infty} \frac{\ln(x)}{e^x})$ L'Hôpital

$= \exp(\lim_{x\to\infty} \frac{1/x}{e^x}) = \exp(0) = e.$